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# Endogenous Exporting Decisions of Heterogeneous Firms: Theory and Evidences

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## Abstract

The paper attempts to explore exporting decisions of heterogeneous firms by endogenously determining both the number of exporting destinations/countries, and the number of varieties for differentiated goods, which is, as we know so far, the first paper to explicitly provide both numbers simultaneously in the literature. Extending from the Melitz's framework (Melitz, 2003), it endogenizes the number of exporting destinations and the number of varieties for differentiated goods for a heterogeneous firm, while providing the cutoff value of transportation cost for the firm's exporting that is different from Melitz's (2003) and other related literatures. Moreover, it shows such numbers are constant at equilibria and they converge to these upper bounds with productivity we find.

In addition, while we show such functions of variety and exporting destinations with respect to productivity are concave, the critique values to ensure the concavity are constant as well and same for both functions, which is surprisingly interesting. Our empirical tests prove theoretical results above as well.

**Keywords:** Heterogeneous firms, productivity, spectrum of exporting destinations, number of differentiated goods

**JEL Classification:** F12, F23

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# **Endogenous Exporting Decisions of Heterogeneous Firms: Theory and Evidences**

## **1. Introduction**

The performance of a heterogeneous firm, especially multinational corporations (i.e. MNCs), on its decision about exporting and/or other decision concerning its exporting has attracted increasing interests widely. For example, people could wonder how many differentiated goods it plans to produce, which should relate to its decision of the number of products. On the other hand, while Melitz (2003) and others initiate the research on heterogeneous firms and show the picture of its decision of production, investment and sales in both domestic and international markets, it opens the door for further research on such heterogeneous firm's decision. For example, when we understand the firm with the highest productivity will choose invest to the oversea to establish its oversea branch, we still donot know how many oversea markets it will choose to sell its product, which should be related with its productivity as well, and those kind of decisions, such as determining the number of exporting markets or destinations and the number of variety of differentiated goods within the same product, is needed to explore more to let people understand more the rationality of firm's behavior. On the

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other hand, however, there is relatively little theoretical research examines how firms determine the spectrum of countries they will export to. That is we expect to explore in this paper.

In relevant literature, one finds that when firms export, they typically export multiple product, they find a positive and significant correlation between the number of products that firms export and the number of countries they export to (Bernard, Jensen, Redding, and Schott, 2007). However, it is only empirical results, without any explicit solution for them. Moreover, they donot explore the relationship and any analytical solution for the number of differentiated goods within one product and the number of exporting destinations, or exporting countries assuming one destination is one country for simplicity in this paper.

Another strand of the literature documents and interprets the relationship between firms' productivity levels and the collection of foreign markets that they serve (Eaton, Kortum, and Kramarz, 2004 and 2007). These papers find that most exporting firms sell to only one foreign market, with the frequency of firms' selling to multiple markets declining with the number of destinations. At the same time, firms selling to only a small number of markets tend to sell to the most popular ones. Less popular markets are served by firms that export very widely. These patterns are consistent with the notion that firms with relatively low marginal costs can profitably exploit relatively more foreign markets.

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Here, we only discuss the condition that all heterogeneous firms in the same industry. So, we can think all the firms produce one kind of product, but there are different brands, as differentiated goods, within the type of product.

The breath of exporting markets is assumed as the number of exporting countries, as we assumed.

## 2. Models

### 2.1. Demand

The preferences of a representative consumer are given by Melitz (2003) as a C.E.S. utility function:

$$U = \int_{\varphi_D}^{\infty} q(\varphi)^\rho d\varphi^{\frac{1}{\rho}}, \quad 0 < \rho < 1,$$

Where  $\varphi$  and  $\varphi_D$  are productivity of the firm's brands and the cutoff productivity of this industry for domestic market. These brands are substitutes, that implies and  $0 < \rho < 1$  an elasticity of substitution between any two brands of  $\sigma = \frac{1}{1-\rho}$ .

Suppose that the producer is also a consumer, this means that a representative consumer has disposable income  $R$ . Where  $R = PQ = \int_{\varphi_D}^{\infty} p(\varphi)q(\varphi)d\varphi$  represents aggregate expenditure of the firm,  $P$  is an aggregate price of the industry.

The consumer attempts to

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$$\max_{q(\varphi)} U \quad \text{s. t.} \quad \int_{\varphi_D}^{\infty} p(\varphi)q(\varphi)d\varphi = R.$$

This provides these following results

$$q(\varphi) = Q \frac{p(\varphi)}{P}^{-\sigma},$$

Where  $Q$  is equal to  $U$ .

$$P = \int_{\varphi_D}^{\infty} p(\varphi)^{1-\sigma} \frac{1}{1-\sigma},$$

$$r(\varphi) = R \frac{p(\varphi)}{P}^{1-\sigma},$$

Where  $R = PQ = \int_{\varphi_D}^{\infty} r(\varphi) d\varphi$  also denotes total revenue of this firm.

## 2.2. Production

Production also attempts to adopt Melitz (2003) as follows: there is only one factor, labor, which is inelasticity supplied. But, here, we divide the labor into two sorts: skilled labor and unskilled workers. Moreover, we assume that organizing the brands requires skilled labor. Production is assumed to be easy and could be accomplished by unskilled workers. Organizing is deemed to be characterized by the diminishing marginal return. Specifically, the organization cost for a certain number  $n$  of brands is  $w \cdot n^m$ ,  $m > 1$ , where  $w$  means each organizer's wage. The  $m$  represents the marginal technology in this country, which is exogenous to firms. To enter the industry each firm has to paying a fixed cost  $F$ , moreover, the first brand's entry cost is  $f_e$  and the sum of  $n$  brands is  $n^m f_e$  for entering the market. The marginal firm is defined as  $n(\varphi_E) = 1$ .

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Unskilled labor function is

$$l = \frac{q}{\varphi},$$

This yields a pricing rule

$$p(\varphi) = \frac{1}{\rho\varphi},$$

Then firm profit is

$$\pi(\varphi) = \frac{R}{\sigma} (P\rho\varphi)^{\sigma-1},$$

That is firm's total revenue minus its total unskilled worker's wage.

An equilibrium will be characterized by a mass of  $\bar{M}$  brands of this type of product in the market and a distribution  $\gamma e^{-\gamma\varphi}$  of productivity levels over a subset of  $(\varphi_D, \infty)$ , where  $\gamma$  is a positive constant.

$$\begin{aligned} P &= \int_{\varphi_D}^{\infty} p(\varphi)^{1-\sigma} \bar{M} \cdot \gamma e^{-\gamma\varphi} \cdot d\varphi^{\frac{1}{1-\sigma}} = \int_{\varphi_D}^{\infty} (\rho\varphi)^{\sigma-1} \bar{M} \cdot \gamma e^{-\gamma\varphi} d\varphi^{\frac{1}{1-\sigma}} \\ &= (B\bar{M})^{\frac{1}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}} \rho^{-1} \end{aligned}$$

Where  $B$  is equal to the improper integral  $\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$  which is constant.

At this moment

$$\pi(\varphi) = \frac{R}{\sigma} (P\rho\varphi)^{\sigma-1} = \frac{R}{\sigma} (B\bar{M})^{\frac{1}{1-\sigma}} \gamma^{\frac{1}{1-\sigma}} \rho^{-1} \rho\varphi^{\sigma-1} = \frac{R\varphi^{\sigma-1}}{B\bar{M}\sigma\gamma}.$$

### 3. Export and its destinations

Given the possible maximum number of exporting markets is  $M$  which this representative firm could have the ability to produce. We also

think the firm treats those countries who it choose to export equally, that is this firm exports the same quality and number of brands to those countries, however, due to different market size of these countries, the exporting quantity maybe different. The marginal condition means  $S(\varphi_E)=0$ , where  $\varphi_E$  is the cutoff productivity for export about these firms, which belong to this industry.

Then, when this firm exports its products, it chooses to produce a certain number of brands  $n$  and spectrum of exporting countries  $S$  for maximization of its profit. That is:

$$\max_{S,n} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) (1+S\tau^{1-\sigma}) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - S\tau n^m f_e - F,$$

Moreover,

$$\begin{aligned} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) \pi_i(\varphi) v(\varphi) d\varphi &= \int_{\varphi_D}^{\varphi} Mu(\varphi) v(\varphi) \sum_{i=1}^n \pi_i(\varphi) d\varphi \\ &= \frac{MR}{B\bar{M}\sigma} \int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi. \end{aligned}$$

So the number of brands is a function of productivity.

Then, the above question can be transformed as the following problem:

$$\max_{S,\varphi} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) (1+S\tau^{1-\sigma}) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - S\tau n^m f_e - F.$$

Through first order conditions of this problem, we can get an analytical expression of  $S$  (see Appendix A):

$$S = \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma\tau f_e} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 - \frac{1}{\tau} \frac{w+f_e}{f_e},$$

Where  $A(\varphi)$  means the definite integral  $\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$ ,  $\varphi^*$  is the

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productivity in equilibrium, which is higher than the cutoff productivity for exporting;  $B$  is equal to the improper integral  $\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$

Where  $\bar{S}$  is a constant in equilibrium

$$\lim_{\varphi \rightarrow \infty} S = \bar{S}.$$

That is, S converges to  $\bar{S}$  with development of productivity and  $\bar{S}$  is the boundary of S.

**Proposition 1** *The spectrum of exporting countries for a firm is the function of its productivity, and there is an upper bound for the spectrum which is constant. Furthermore, with development of its productivity, the spectrum would approach to this boundary finally.*

*Comparative statics:*

$$\frac{\partial S}{\partial w} = \frac{1}{\tau f_e} \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma\tau^\sigma} - 1 > 0,$$

$$\frac{\partial S}{\partial R} = \frac{M \cdot A(\varphi^*)}{B\bar{M}\sigma\tau f_e} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 > 0,$$

$$\frac{\partial S}{\partial \tau} = \frac{1}{\tau^2} \frac{w+f_e}{f_e} \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma f_e} \frac{w+f_e}{f_e}^{-1} - \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma\tau^\sigma f_e} - 1 - \frac{MR \cdot A(\varphi^*)}{B\bar{M} f_e},$$

*Then*

$$\frac{\partial S}{\partial \tau} < \frac{1}{\tau^2} \frac{w+f_e}{f_e} - 0 - \frac{MR \cdot A(\varphi^*)}{B\bar{M} f_e} < 0,$$

$$\frac{\partial S}{\partial f_e} = \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma\tau f_e^2} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 + \frac{w}{\tau f_e^2} \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma\tau^\sigma f_e} - 1 < 0.$$

**Remark:** The raise of average wage can absorb more excellent employee for a firm, which brings the development of its technology and organization, then its revenue increases, all these will induce its profit and

spectrum of exporting countries to rise, so both partial derivatives of spectrum with respect to average wage and total revenue are all positive. But the increase of transaction cost and the first brand's entry cost induce contrary change in its profit, reducing its exporting spectrum.

The curve of  $S = S(\varphi)$ :

$$S'(\varphi) = \frac{MR \cdot \varphi^{\sigma-1} e^{-\gamma\varphi}}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w + f_e}{f_e} > 0,$$

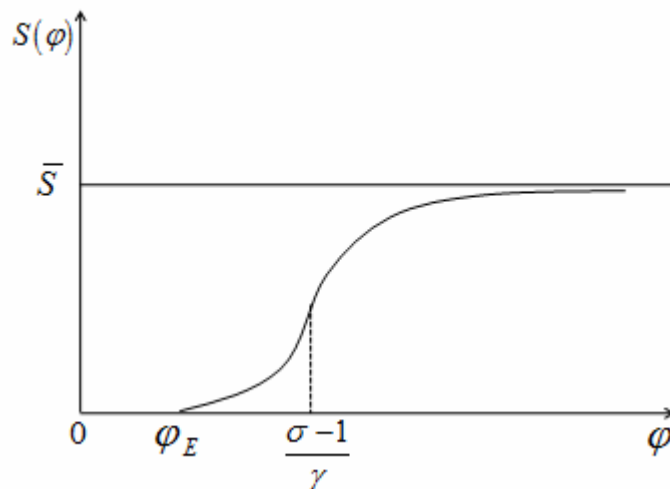
This means the spectrum of exporting countries increases with productivity.

$$S''(\varphi) = \frac{\sigma-1}{\gamma} - \varphi \frac{MR \cdot \varphi^{\sigma-1} e^{-\gamma\varphi}}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w + f_e}{f_e},$$

$$S''(\varphi) > 0, \quad \varphi_E \leq \varphi \leq \frac{\sigma-1}{\gamma} \quad \text{convex};$$

$$S''(\varphi) < 0, \quad \varphi > \frac{\sigma-1}{\gamma}, \quad \text{concave}.$$

Where  $\varphi_E$  is the exporting cutoff productivity for this industry and we assume that  $\varphi_E$  is less than  $\frac{\sigma-1}{\gamma}$ .



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Figure1 The shape of curve  $S = S(\varphi)$

Then we have

**Proposition 2** *There is a value  $\frac{\sigma-1}{\gamma}$ , if the productivity level is between*

*$\varphi_E$  and  $\frac{\sigma-1}{\gamma}$ , the curve of  $S = S(\varphi)$  is convex; if higher than it, the*

*curve is concave.*

*Average wage of the firm:*

$$\bar{w} = \lambda_1 \cdot 1 + \lambda_2 w, \quad \lambda_1 \in (0,1), \lambda_2 \in (0,1),$$

*Then*

$$\frac{\partial S}{\partial \bar{w}} = \frac{\partial S}{\partial w} \frac{\partial w}{\partial \bar{w}} = \frac{1}{\lambda_2} \frac{\partial S}{\partial w} > 0 \quad \lambda_2 \in (0,1).$$

*Where  $\lambda_1$  and  $\lambda_2$  are the proportions of unskilled and skilled labor to the firm's total workers respectively.*

**Remarks:** From the above, we can see that the spectrum of exporting countries also increases with average wage of this firm.

#### 4. Variety of differentiated goods

Firm entry and exit

The marginal condition is defined as that a firm only produces one kind of product ( $n(\varphi_D) = 1$ ) and whose net profit after organization cost is just enough to cover the fixed cost  $F$ .

Then the zero profit conditions are given as the following under different situations:

Closed economy  $\pi_1(\varphi_D) = F + w,$

Export  $\pi_1(\varphi_E) = F + w + \tau f_e,$

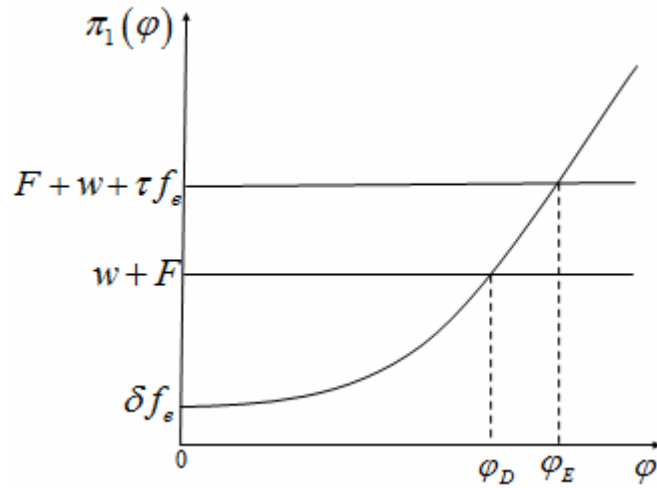


Figure 2 Zero profit condition

Where  $\varphi_E$  is the cutoff productivity of export firms for the industry.

Free entry condition

$$\frac{1-G(\varphi)}{\delta} \pi_1 - f_e = 0,$$

$$\pi_1 = \frac{\delta f_e}{1-G(\varphi)},$$

Where  $1-G(\varphi_D)$  is the ex ante probability of drawing a productivity above the zero-value cutoff  $\varphi_D$  upon entry into the industry. When the firm does produce,  $\delta$  is a constant probability which the firm faces in every period of a bad shock that would force it to exit.

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## 4.1 Closed Economy

As we discussed before, the representative firm expects to maximize its profit with respect to variety of this firm:

$$\max_n \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - F,$$

Where  $u(\varphi)v(\varphi)$ , being equal to  $\gamma e^{-\gamma\varphi}$ , is the probability of drawing productivity  $\varphi$  upon entry into the market.

Moreover,

$$\begin{aligned} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) \pi_i(\varphi) v(\varphi) d\varphi &= \int_{\varphi_D}^{\varphi} Mu(\varphi) v(\varphi) \sum_{i=1}^n \pi_i(\varphi) d\varphi \\ &= \frac{MR}{B\bar{M}\sigma} \int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi. \end{aligned}$$

So the variety is a function of productivity. The problem of maximization of profit with respect to variety can be transformed the following issue:

$$\max_{\varphi} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - F,$$

Then we obtain the following result (see Appendix D for proof)

$$n = \frac{MR \cdot A(\varphi^*)}{B\bar{M}\sigma \cdot (w + f_e)} + 1^{\frac{1}{m}}$$

Where  $A(\varphi)$  means the definite integral  $\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$ ,  $\varphi^*$  is the productivity in equilibrium.

Comparative statics:

$$\frac{\partial n}{\partial m} = -\frac{1}{m^2} \frac{MR \cdot A(\varphi)}{B\bar{M}\sigma \cdot (w + f_e)} + 1^{\frac{1}{m}} \ln \frac{MR \cdot A(\varphi)}{B\bar{M}\sigma \cdot (w + f_e)} + 1 < 0,$$

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For a certain firm, larger marginal technology  $m$  would lead to higher organizational cost, which reduces the firm's profit gain and then the quantity of its number of varieties.

$$\frac{\partial n}{\partial w} = -\frac{MR}{m\sigma \cdot (w + f_e)^2} \frac{MR \cdot A(\varphi)}{BM\sigma \cdot (w + f_e)} + 1 \frac{\frac{1}{m}-1}{BM} \frac{A(\varphi)}{BM} < 0,$$

$$\frac{\partial n}{\partial f_e} = -\frac{MR}{m\sigma \cdot (w + f_e)^2} \frac{MR \cdot A(\varphi)}{BM\sigma \cdot (w + f_e)} + 1 \frac{\frac{1}{m}-1}{BM} \frac{A(\varphi)}{BM} < 0.$$

Similarly, an increase in the unskilled labor wage or the first brand's entry cost of a firm would also result in higher total cost and then decline of the firm's varieties.

## 4.2 Open Economy

We assume that the world is comprised of  $S+1$  identical countries. The transaction cost  $\tau$  is the standard iceberg formulation, where  $\tau > 1$  units of a good must be shipped in order for 1 unit to arrive at destination.

Maximization of its profit

$$\max_{\varphi} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) (1 + S\tau^{1-\sigma}) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - S\tau n^m f_e - F,$$

Where  $M$ : the possible maximum number of varieties, it can be given as a Constant;  $u(\varphi)$  and  $v(\varphi)$ : the probability distribution functions which draw from the possible maximal varieties and productivity respectively, two steps would be taken here, firstly a few is drew out from the possible maximal varieties, secondly, by  $v(\varphi)$ , some productivity is drew out for

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producing;  $f_e$ : the first variety's entry cost in the domestic market,  $n^m f$ :  
the total entry cost of a firm with  $n$  kinds of product in closed economy;

$wn^m$

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$$\frac{d^2 n}{d\varphi^2} = (1-m) \frac{\frac{MR \cdot (1+S\tau^{1-\sigma})^{\sigma-1} \varphi^{\sigma-1} e^{-\gamma\varphi}}{BM\sigma \cdot w+(1+S\tau)f_e}}{m^2 \cdot \frac{MR \cdot (1+S\tau^{1-\sigma})A(\varphi)}{BM\sigma \cdot w+(1+S\tau)f_e} + 1} + \frac{\sigma-1}{\varphi} - \gamma \frac{MR \cdot (1+S\tau^{1-\sigma}) \varphi^{\sigma-1} e^{-\gamma\varphi}}{BMm\sigma \cdot w+(1+S\tau)f_e \cdot \frac{MR \cdot (1+S\tau^{1-\sigma})A(\varphi)}{BM\sigma \cdot w+(1+S\tau)f_e} + 1}.$$

Then, if the productivity is higher than  $\frac{\sigma-1}{\gamma}$ ,  $\frac{\partial^2 n}{\partial \varphi^2}$

$$\frac{\partial n}{\partial m} = - \frac{\ln \frac{AMR \cdot (1 + S\tau^{1-\sigma})}{B\bar{M}\sigma \cdot w + (1 + S\tau) f_e} + 1}{\frac{AMR \cdot (1 + S\tau^{1-\sigma})}{B\bar{M}\sigma \cdot w + (1 + S\tau) f_e} + 1} \frac{1}{m^2} < 0$$

$$\frac{\partial n}{\partial \tau} = - \frac{AMR \cdot \{(\sigma - 1)S\tau^{-\sigma} w + (S\tau + 1) f_e + (1 + S\tau^{1-\sigma}) S f_e\}}{B\bar{M}\sigma m \cdot w + (S\tau + 1) f_e^2} \frac{1}{m^{1-\frac{1}{m}}} < 0,$$

When exporting to other countries, an increase in transaction cost  $\tau$  can bring higher total cost to a firm, this reduces the firm's profit, so the number of brands will become fewer.

$$\frac{\partial n}{\partial w} = - \frac{\frac{AMR \cdot (1 + S\tau^{1-\sigma})}{B\bar{M}\sigma m \cdot w + (1 + S\tau) f_e^2}}{\frac{AMR \cdot (1 + S\tau^{1-\sigma})}{B\bar{M}\sigma \cdot w + (1 + S\tau) f_e} + 1} \frac{1}{m} < 0$$

Average wage for the firm:

$$\bar{w} = \lambda_1 \cdot 1 + \lambda_2 w, \quad \lambda_1 \in (0, 1), \lambda_2 \in (0, 1),$$

Then

$$\frac{\partial \bar{w}}{\partial w} = \frac{\partial n}{\partial w} \frac{\partial w}{\partial w} = \frac{1}{\lambda_2} \frac{\partial n}{\partial w} < 0 \quad \lambda_2 \in (0, 1).$$

Where  $\lambda_1$  and  $\lambda_2$  are the proportions of unskilled and skilled labor to the firm's total workers respectively.

$$\frac{\partial n}{\partial S} = \frac{\frac{AMR\tau^{1-\sigma} \cdot w + f_e - \tau^\sigma f_e}{B\bar{M}\sigma m \cdot w + (S\tau + 1) f_e^2}}{\frac{AMR \cdot (1 + S\tau^{1-\sigma})}{B\bar{M}\sigma \cdot w + (1 + S\tau) f_e} + 1} \frac{1}{m-1}$$

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$$\frac{\partial n}{\partial S} \geq 0, \quad 1 < \tau \leq \frac{w + f_e}{f_e}^{\frac{1}{\sigma}},$$

$$\frac{\partial n}{\partial S} < 0, \quad \tau > \frac{w + f_e}{f_e}^{\frac{1}{\sigma}}.$$

**Proposition 4:** *When a firm exports, there is a value  $\frac{w + f_e}{f_e}^{\frac{1}{\sigma}}$ , if the transaction cost is lower than it, the number of varieties increase with exporting countries; if higher than it, the number would decrease.*

For proposition 4, the existing value  $\frac{w + f_e}{f_e}^{\frac{1}{\sigma}}$  is determined by skilled labor's wage, first variety's entry cost and elasticity of substitution between any two goods. But, it is a constant in the equilibrium, because the average wage would be a constant at that time.

If the transaction cost is lower than it, all the exporting varieties are profitable, then the firm has more richer capital to introduce new varieties, so the number of total product varieties would increase with exporting countries; if higher than it, a few of exporting varieties would be losing gradually. Moreover, with the increase of exporting countries the competition would become fiercer, particularly the average wage has a trend of increase with time, all these factors result in the reduction of number.

## 5. Empirical tests

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## 5.1 Exporting destinations

Assume that the first variety's entry cost and the transaction cost are all constant. We can get the empirical model

$$S_{it} = \beta_1 R_{it} + \beta_2 \varphi_{it} + \beta_3 \bar{w}_{it} + u_i + v_{it},$$

Where  $\varphi$  is measured in terms of either revenue-based labor productivity or TFPR, here we choose revenue-based measures of productivity, but, not quantity-based measures of productivity<sup>3</sup>, because data on physical units of output is not available for all products, moreover, physical units are not comparable across firms for many products, e.g., cars, motorcycles etc; company is denoted by  $i$  and  $t$  means year.  $u_i$  captures the company fixed effects and  $v_{it}$  is a stochastic error.

The dataset in this research is constructed for 74 motorcycle firms of China for 7-year period from 2000 to 2006. This 7 years period is between Asian financial crisis in 1997 and global financial crisis in 2007 and in this period China's overall export shows no dramatic variation.

Figure 4 illustrates that total spectrum of exporting countries per year is 64,108,113,135,149,158 and 170 respectively in the period of 2000 to 2006. The spectrum of 2006 is almost triplication of 2000 and there is very large rise between 2000 and 2001.

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<sup>3</sup> Both revenue- and quantity-based measures of productivity are monotonically related to the firm productivity drawn (Bernard, Redding and Schott, 2010.), and there is a positive correlation between them (Foster, Haltiwanger

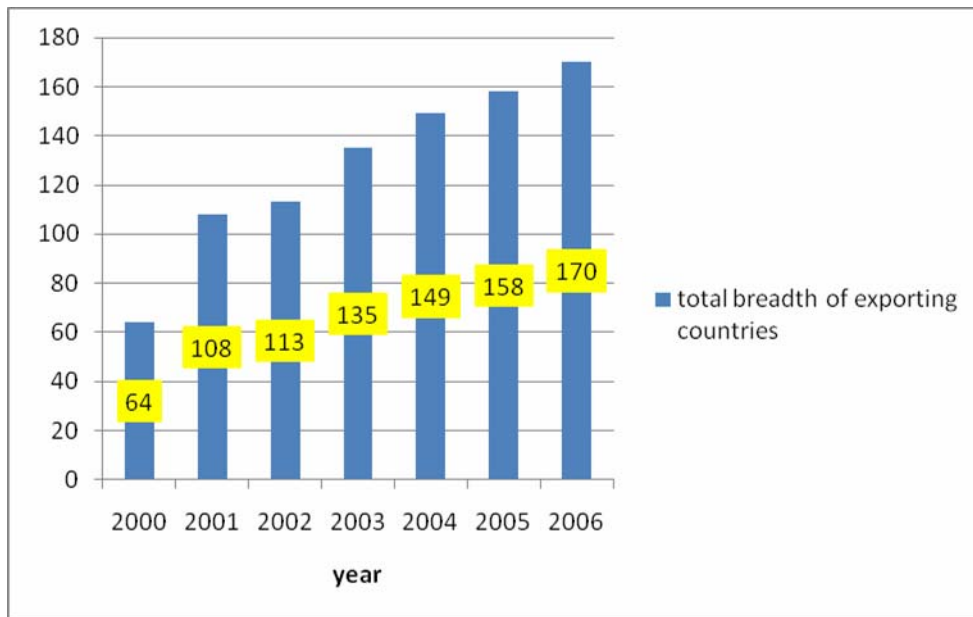


Figure 4: Total spectrum of exporting countries per year

Table 1 Descriptive statistics

Variables	Mean	SD	Min	Max
$S$	16.85	16.59571	1	83
$R$	25240.7	430875.8	3486	3641329
$\varphi$	499.5289	506.7539	11.97716	4422.953
$\bar{w}$	14.48639	8.23511	2.66187	54.955

Data source: National Bureau of Statistics of China and General Administration of Customs of the People's Republic of China.

In Table 1, the units of total revenue  $R$ , productivity  $\varphi$ , average wage  $\bar{w}$  and spectrum of exporting countries  $S$  of a firm are thousand RMB, thousand RMB per labor, thousand RMB per person and number of export countries respectively. Here total revenue is the sum of sales

revenue of a firm and its subsidy received from the government. Average wage is total wage which equals that actual wage plus welfare provided by the firm divided by the number of staff.

As to panel data estimations, the Hausman test (see the note of Table 2) for the difference between the Fixed-effects and Random-effects is not significant, suggesting that a Random-effect model is preferred over Fixed-effect model, so Random-effect method is employed.

Table 2 General estimates

Variables	Estimation model		
	(1)	(2)	(3)
$R$	6.42e-06** (2.15)	6.40e-06** (2.12)	
$\varphi$	0.0080474*** (3.73)	0.0080388*** (3.70)	
$\bar{w}$	0.079162 (0.67)	0.0793396 (0.66)	
INTERCEPT	9.643431*** (4.67)	9.645038*** (4.62)	
Estimation method	MLE	RE	GMM
R-square		0.1787	
Observations	260	260	

Results are based on data of 74 motorcycle companies for 7 year period from 2000 to 2006 in China. The diagnostics tests results include: Hausman test:  $\chi^2(2)=0.07$ ,  $p = 0.9644$ ; Wooldridge test for

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autocorrelation:  $F(1,51) = 13.868$ ,  $p < 0.00$  ; White's heteroskedasticity test:  $\chi^2(9) = 21.04$ ,  $p < 0.05$ .  $t$ -statistics in parentheses, and significant at 1% and 5% level respectively.

Furthermore, Wooldridge test for autocorrelation in panel data and modified Wald test for heteroskedasticity (see the note of Table 2) show that there is a significant first-order autocorrelation and heteroskedasticity within enterprises for the panel dataset.

In Table 2, these results of all three estimations show that: these three coefficients are all positive, supporting the above conclusions of comparative statistics in theoretical part; both total revenue and productivity of a firm have significant (revenue is at 5% level, productivity at 1% level.) impact on its spectrum of exporting countries; however, the effect of average wage isn't significant. For average wage of a firm, its raise can absorb more excellent employee, however, at the same time this also results in higher cost and less revenue, which brings the reduction of export quantity and spectrum of exporting countries.

## 5.2 Variety of differentiated goods

Number of export varieties

$$n_E = \frac{A_E MR \cdot (1 + S\tau^{1-\sigma})}{B_E M \sigma \cdot w + (1 + S\tau) f_e} + 1^{\frac{1}{m}},$$

Where  $A_E, B_E$  are equal to  $\int_{\varphi_E}^{\varphi^*} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$  and  $\int_{\varphi_E}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi$  at this time.

Then

$$\begin{aligned} \frac{\partial n_E}{\partial w} < 0, & \quad \frac{\partial n_E}{\partial \tau} < 0, & \quad \frac{\partial n_E}{\partial f_e} < 0, \\ \frac{\partial n_E}{\partial S} \geq 0, & & \quad 1 < \tau \leq \frac{w + f_e}{f_e}^{\frac{1}{\sigma}}, \\ \frac{\partial n_E}{\partial S} < 0, & & \quad \tau > \frac{w + f_e}{f_e}^{\frac{1}{\sigma}}. \end{aligned}$$

Assume that the marginal technology, the first variety's entry cost and the transaction cost are all constants. We can get the empirical model

$$n_{Eit} = \alpha_1 R_{it} + \alpha_2 \varphi_{it} + \alpha_3 \bar{w}_{it} + \alpha_4 S_{it} + u_i + v_{it},$$

Where  $\varphi_{it}$  is measured in terms of either revenue-based labor productivity or TFPR; company is denoted by  $i$  and  $t$  means year;  $u_i$  captures the company fixed effects and  $v_{it}$  is a stochastic error.

Table 3 Descriptive statistics

Variables	Mean	SD	Min	Max
$R$	5779290	1.09e+07	0	5.89e+07
$\varphi$	1067.232	1437.54	0	10291.55
$\bar{w}$	32.4892	26.01499	0	169.2304
$S$	11.21818	14.07041	0	66
$n$	4.036364	5.138201	0	38

Data source: Data source: National Bureau of Statistics of China and General Administration of Customs of the People's Republic of China.

The units of  $R, \varphi, \bar{w}, S$  and  $n$  are thousand RMB, thousand RMB per labor, thousand RMB per person, number of export countries and number of heterogeneous varieties respectively. The value of zero means that the firm still isn't in business for that year.

Through Wooldridge test, autocorrelation exists in the panel data. With the result of Hausman test, the fixed method is be used here.

Table 4 General estimates

Variables	Fixed effect method	
$R$	1.59e-07***	(3.45)
$\varphi$	0.0001913	(0.39)
$\bar{w}$	-.0103878	(-0.47)
$S$	0.2266864***	(10.56)
INTERCEPT	0.7061579	(1.09)
R-square	0.6471	
Observations	165	

Results are based on data of 33 automobile companies for 5 year period from 2001 to 2006 in China. The diagnostics tests results include: Hausman test  $\chi^2(3)=12.99, p < 0.01$ ; Wooldridge test for autocorrelation:  $F(1,32) = 0.064, p = 0.8017$ ; White's heteroskedasticity test:

$\chi^2(14)=82.27, p < 0.00$ .  $t$ -statistics in parentheses, and significant at 1% and 5% level

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respectively.

The signs are the expected ones. But the effects of average wage and productivity aren't significant. We provide explanations as the following:

For the number of export varieties, you know that China has entered into WTO from 2001. We may think there is a sharp decline in the transaction cost so that it is lower than the value  $\frac{w+f_e}{f_e}^{\frac{1}{\sigma}}$ . Then the coefficient of  $S$  would be positive.

For the average wage, we consider that it hasn't any intimate relationship with export varieties that requires higher level productivity which is closer to skilled labor, so we think its effect on the change of export variety isn't significant.

As far as the effect of productivity, one reason is from the following curve: if there is large growth on productivity, however, as can be seen from the graph, it only results in a very tiny change of exporting variety.

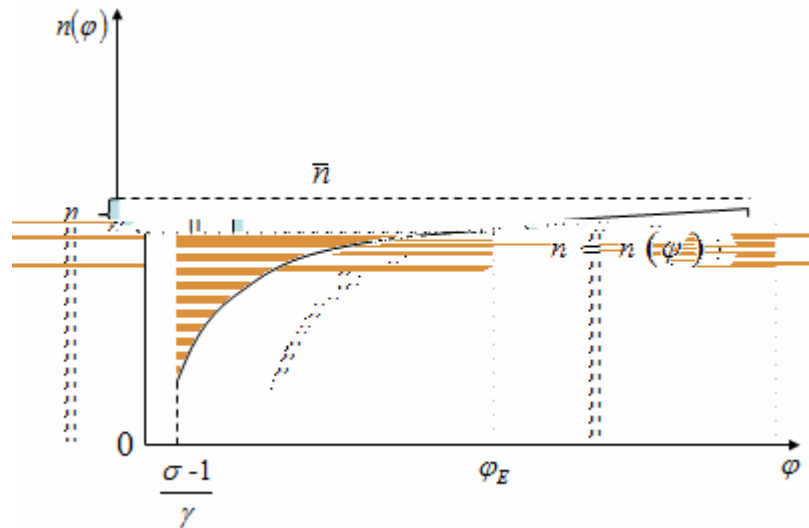


Figure 5 The interpretation of productivity's significance

Another is that: for Chinese automobile industry, the development of export variety is very slowly, the vast majority of its varieties are produced for the domestic market. From these two explanations we can see that the effect of productivity is also not significant.

## 6. Concluding Remarks

We adopt Melitz model (2003) and obtain the close-form solution of numbers of variety of differentiated goods and exporting markets, respectively. That is, one could know the number of exporting destination is fixed as long as the MNC achieves its maximum of its profit. Moreover, one also knows the trend of change of such number as the productivity grows. Furthermore, the number of differentiated goods could be

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determined by the firm simultaneously as the number of exporting markets as we shows above. The empirical test also confirms all of our results. It should extend current understanding for the performance of MNCs

On the other hand, this paper shows the number is a constant in equilibrium and then gives the upper bound of this number which is constant, moreover, we claims that the number of total varieties converge to this upper bound with productivity, but cannot reach this bound all the time.

The paper also proves that the function of variety with respect to productivity is concave.

For the exporting firms, we get the conclusion: there is a value which is determined by skilled labor's wage, first variety's entry cost and elasticity of substitution between any two goods., if the transaction cost is lower than it, the number of varieties increase with exporting countries; if higher than it, number would decrease.

At last, through empirical test, we know the signs of these four coefficients of revenue, productivity, average wage and number of exporting varieties are consistent with theoretical expectations. The firm's revenue and number of exporting countries have significant effects on the change of export variety, however, the effects of productivity and average wage are not significant, then the paper provides reasonable explanations

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of these empirical results with the discussion of theoretical part and the fact of Chinese automobile industry.

## Appendices

### Appendix A:

Maximization of profit

$$\max_{S, \varphi} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) (1 + S\tau^{1-\sigma}) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - S\tau n^m f_e - F,$$

First order conditions:

Given  $S$  :

$$\sum_{i=1}^n Mu(\varphi) (1 + S\tau^{1-\sigma}) \pi_i(\varphi) v(\varphi) - m w + (1 + s\tau) f_e n^{m-1} n'(\varphi) = 0,$$

$$n^{m-1} n'(\varphi) = \frac{MR \cdot (1 + S\tau^{1-\sigma})}{m w + (1 + s\tau) f_e} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{M\sigma \int_{\varphi_D}^{\infty} \varphi^{1-\sigma} e^{-\gamma\varphi} d\varphi},$$

Where  $u(\varphi)v(\varphi)$  is equal to  $\gamma e^{-\gamma\varphi}$ .

$$\frac{1}{m} n^m = \frac{MR \cdot (1 + S\tau^{1-\sigma})}{Mm\sigma \cdot w + (1 + s\tau) f_e} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + b,$$

Where  $b$  is constant.

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$$n^m = \frac{MR \cdot (1 + S\tau^{1-\sigma})}{BM\sigma \cdot w + (1 + S\tau)f_e} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + mb.$$

And  $n(\varphi_D) = 1$ , then we yield

$$1 = n(\varphi_D)^m = n^m = \frac{MR \cdot (1 + S\tau^{1-\sigma})}{BM\sigma \cdot w + (1 + S\tau)f_e} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + mb,$$

We get

$$mb = 1,$$

So

$$n^m = \frac{MR \cdot (1 + S\tau^{1-\sigma}) A(\varphi)}{BM\sigma \cdot w + (1 + S\tau)f_e} + 1.$$

Given  $\varphi$ :

$$\frac{MR\tau^{1-\sigma} \cdot A(\varphi)}{BM\sigma} - \tau n^m f = 0,$$

Then

$$S = \frac{MR \cdot A(\varphi^*)}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w + f_e}{f_e} - 1 - \frac{1}{\tau} \frac{w + f_e}{f_e}.$$

## Appendix B:

$$S = \frac{MR \cdot A(\varphi^*)}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w + f_e}{f_e} - 1 - \frac{1}{\tau} \frac{w + f_e}{f_e} \geq 0,$$

so

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$$\frac{MR \cdot A(\varphi^*)}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 \geq \frac{1}{\tau} \frac{w+f_e}{f_e} > 0,$$

Moreover,  $R$  and  $A(\varphi^*)$  are positive, then we obtain

$$\tau^{-\sigma} \frac{w+f_e}{f_e} - 1 > 0,$$

That is

$$\tau < \frac{w+f_e}{f_e}^{\frac{1}{\sigma}}.$$

### Appendix C:

$$S = \frac{MR \cdot A(\varphi^*)}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 - \frac{1}{\tau} \frac{w+f_e}{f_e} \geq 0,$$

So

$$\frac{MR \cdot A(\varphi^*)}{BM\sigma\tau f_e} \tau^{-\sigma} \frac{w+f_e}{f_e} - 1 \geq \frac{1}{\tau} \frac{w+f_e}{f_e},$$

$$\frac{MR \cdot A(\varphi^*)}{BM\sigma f_e \tau^\sigma} - 1 \geq \frac{MR \cdot A(\varphi^*)}{BM\sigma f_e} \frac{w+f_e}{f_e}^{-1},$$

Moreover,  $R$  and  $A(\varphi^*)$  are positive, then we obtain

$$\frac{MR \cdot A(\varphi^*)}{BM\sigma f_e \tau^\sigma} - 1 \geq \frac{MR \cdot A(\varphi^*)}{BM\sigma f_e} \frac{w+f_e}{f_e}^{-1} > 0.$$

### Appendix D:

Profit Maximization

$$\max_{\varphi} \sum_{i=1}^n \int_{\varphi_D}^{\varphi} Mu(\varphi) \pi_i(\varphi) v(\varphi) d\varphi - wn^m - n^m f_e - F, \quad (5)$$

First order condition

$$\sum_{i=1}^n Mu(\varphi) \pi_i(\varphi) v(\varphi) - m(w + f_e) n^{m-1} n'(\varphi) = 0,$$

$$n^{m-1} n'(\varphi) = \frac{MR}{m(w + f_e) M\sigma} \frac{\varphi^{\sigma-1} e^{-\gamma\varphi}}{\int_{\varphi_D}^{\infty} \varphi^{1-\sigma} e^{-\gamma\varphi} d\varphi},$$

Where  $u(\varphi)v(\varphi)$  is equal to  $\gamma e^{-\gamma\varphi}$ .

$$\frac{1}{m} n^m = \frac{MR}{M\sigma \cdot (w + f_e)} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + b,$$

Where  $b$  is constant.

$$n^m = \frac{MR}{M\sigma \cdot (w + f_e)} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + mb.$$

And  $n(\varphi_D) = 1$ , then we yield

$$1 = n(\varphi_D)^m = n^m = \frac{MR}{M\sigma \cdot (w + f_e)} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + mb,$$

We get

$$mb = 1,$$

So

$$n = \frac{MR}{M\sigma \cdot (w + f_e)} \frac{\int_{\varphi_D}^{\varphi} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi}{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} e^{-\gamma\varphi} d\varphi} + 1^{\frac{1}{m}}$$

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