

# Does mortality decline always lead to an increase in human capital investment?

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## Abstract

According to conventional wisdom in human capital theory, mortality decline leads to more schooling, because individuals expect to have more years to reap the benefits of human capital investment. This result is shown, analytically or computationally, by many researchers. However, the researchers usually represent mortality reductions by changes in survival parameters, with a parameter change causing mortality changes in different ages simultaneously. We use a general non-parametric approach to the survival function, and apply Volterra derivative for a functional to consider the effect of mortality decline at an arbitrary age. This approach enables us to show that mortality reductions at younger versus older ages have systematically different effects on optimal schooling years. In particular, a mortality decline during the studying phase leads to shorter schooling years, providing a counter-example to the conventional wisdom.

**JEL Classification Numbers:** I20; J10; J24

**Keywords:** mortality decline; human capital investment; retirement age

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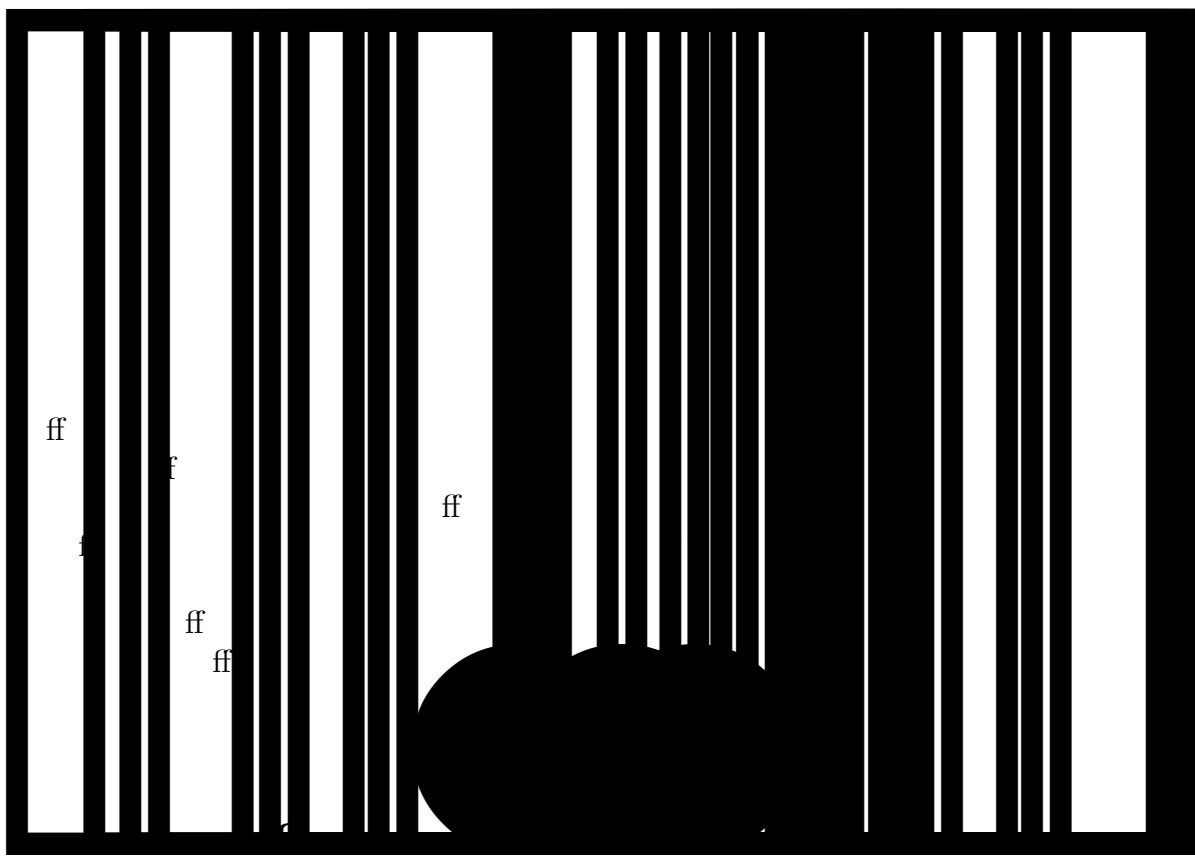
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$$\begin{array}{l} \mathsf{I}(\mathsf{x}) \\ \mu(\mathsf{q}) \geq 0 \\ \mathsf{q} \end{array} \qquad \begin{array}{l} \mathsf{I}(\mathsf{x}) = \mathsf{e}^{-\int_0^{\mathsf{x}} \mu(q) dq}, \\ \mathsf{I}(0) = 1 \quad \mathsf{I}(\mathsf{T}) = 0 \\ \lim_{q \rightarrow \mathsf{T}} \mu(\mathsf{q}) = \infty \end{array}$$

$$\begin{array}{ccccc} & & \mathsf{S} & & \mathsf{R} \\ \hline \mathsf{fl} & & & & \mathsf{fi} \\ & & a'(x) = [r + \mu(x)]\,a(x) - \phi(x) - c(x) & & \\ x \leq S & \phi(x) & & x & \mathsf{fi} \end{array}$$

$$h'\left(S^*\right)\left[\int_S^R e^{-rx}l\left(x\right)w\left(x\right)dx\right]=e^{-rS}\,l\left(S^*\right)\left[h\left(S^*\right)w\left(S^*\right)+\phi\left(S^*\right)\right].$$

$$\int_0^T e^{-\rho x} l\left( x\right) \frac{c\left( x\right) ^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}dx-\int_0^R e^{-\rho x} l\left( x\right) f\left( x\right) dx,$$

$$a'\left( x\right) =\left\{ \begin{array}{l} \left[ r+\mu \left( x\right) \right] a\left( x\right) +h\left( S\right) w\left( x\right) -$$

$$\widetilde{R}(S) \qquad \qquad S \qquad \qquad \text{fi}$$

$$\mathrm{e}^{-r\widetilde{R}(S)}h(S)w\left(\widetilde{R}(S)\right)c\left(0,S,\widetilde{R}(S)\right)^{-\frac{1}{\sigma}}=\mathrm{e}^{-\rho\widetilde{R}(S)}f\left(\widetilde{R}(S)\right).$$

$$\widetilde{R}(S) \qquad \qquad \text{fi}$$

$$h'(S\,)\,\left[\int_{S^*}^{R^*}e^{-rx}l(x)w(x)\,dx\right]=e^{-rS^*}l(S\,)\,h(S\,)w(S\,),$$

$$R\,=\widetilde{R}(S\,)\,,$$

$$\text{fi}$$

$$\text{fi}$$

$$S$$

$$S$$

$$R$$

$$S$$

$$\text{ff}$$

$$\mathbf{x}_0$$

$$\text{fi}$$

$$\frac{-\partial l\left(\mathbf{x}\right)}{\partial \mu\left(\mathbf{x}_0\right)}=\left\{\begin{array}{ll}0&\mathbf{x}<\mathbf{x}_0\\l\left(\mathbf{x}\right)&\mathbf{x}\geq\mathbf{x}_0\end{array}\right.$$

$$\mathbf{x}_0$$

$$\mathbf{x}_0$$

$$\text{ff}$$

$$\hline$$

$$e^{-R}l(R)f(R)$$

$$R$$

$$l(R)f(R)$$

$$\text{fi}$$

$$e^{-rR}l(R)h(S)w(R)$$

$$c(0,S,R)^{-\frac{1}{\sigma}}\frac{-l(x)}{C(x_\theta)}$$

$$\text{ff}$$

$$x$$

$$x_0\qquad (x)$$



$$\begin{array}{ccccc} & & \text{ff} & & \text{S} \\ & & & & \\ \text{fi} & & \text{I}(\text{S}) & & \text{fi} \\ & & & & \text{ff} \end{array}$$

$$\begin{array}{ccccc} & & \text{ff} & & \text{fi} \\ & & & & \\ & & & & \infty \end{array}$$

$$\frac{-\partial \tilde{R}(S^*,\mu(.))}{\partial \mu(x_0)} \cdot \left[ \text{sign} \frac{-\partial \text{S}}{\partial \mu(x_0)} \right] = \text{sign} \frac{\int_{S^*}^{R^*} e^{-rx} \left( \frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) \, dx}{\int_{S^*}^{R^*} e^{-rx} l(x) w(x) \, dx} - \frac{\left( \frac{-\partial l(S^*)}{\partial \mu(x_0)} \right)}{l(S^*)} \, .$$

$$\int_{S^*}^{R^*} e^{-rx} \left( \frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) \, dx = \begin{array}{cc} \int_{S^*}^{R^*} e^{-rx} l(x) w(x) \, dx & x_0 \leq S \\ \int_{x_0}^{R^*} e^{-rx} l(x) w(x) \, dx & S < x_0 < R \\ 0 & x_0 \geq R \end{array} \, ,$$

$$\frac{-\partial l(S^*)}{\partial \mu(x_0)} = \left\{ \begin{array}{cc} l(S^*) & x_0 \leq S \\ 0 & x_0 > S \end{array} \right. \, .$$

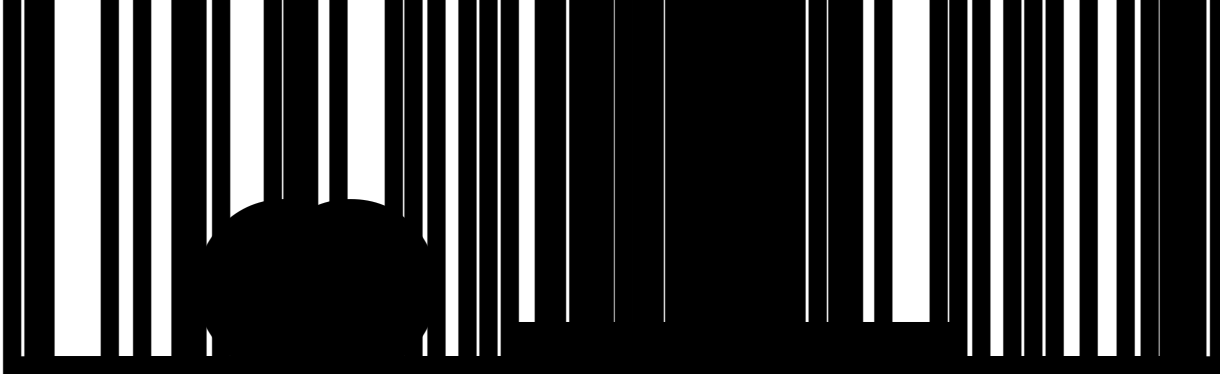
$$\text{sign} \frac{\int_{S^*}^{R^*} e^{-rx} \left( \frac{-\partial l(x)}{\partial \mu(x_0)} \right) w(x) \, dx}{\int_{S^*}^{R^*} e^{-rx} l(x) w(x) \, dx} - \frac{\left( \frac{-\partial l(S^*)}{\partial \mu(x_0)} \right)}{l(S^*)} = \begin{array}{cc} 0 & x_0 \leq S \\ +1 & S < x_0 < R \\ 0 & x_0 \geq R \end{array} \, .$$

ff

S

R

$$x_0 \leq S$$



fi

$$\text{ff} \quad \frac{e^{-rR^*}l(R^*)w(R^*)}{\int_{S^*}^{R^*}e^{-rx}l(x)w(x)dx}$$

$$\frac{-\partial \tilde{R}(S,\mu(.))}{\partial \mu(x_0)}$$

$$\text{sign}\left[\frac{-\partial \tilde{\mathbf{R}}\left(\mathbf{S},\mu\left(\cdot\right)\right)}{\partial \mu\left(\mathbf{x}_0\right)}\right]=-\text{sign}\;\frac{-\partial \mathfrak{c}\left(0,\mathbf{S},\tilde{\mathbf{R}}\left(\mathbf{S},\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}{\partial \mu\left(\mathbf{x}_0\right)}\;,$$

$$\mathfrak{c}^{-\frac{1}{\sigma}}\\ \text{fi}$$

fi

ff

$$\mathbf{S}=\mathbf{S}$$

fi

$$\begin{array}{c} \frac{1}{\mathfrak{c}\left(0,\mathbf{S}\;,\tilde{\mathbf{R}}\left(\mathbf{S}\;,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}\;\frac{-\partial \mathfrak{c}\left(0,\mathbf{S}\;,\tilde{\mathbf{R}}\left(\mathbf{S}\;,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}{\partial \mu\left(\mathbf{x}_0\right)}\\[10pt] =\frac{1}{\mathbf{H}\left(\mathbf{S}\;,\tilde{\mathbf{R}}\left(\mathbf{S}\;,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}\;\frac{-\partial \mathbf{H}\left(\mathbf{S}\;,\tilde{\mathbf{R}}\left(\mathbf{S}\;,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}{\partial \mu\left(\mathbf{x}_0\right)}\\[10pt] \hline \mu\left(\cdot\right) \qquad \qquad m\left(0\right)\;-\;H\left(0,S,R\right) \qquad \qquad c\left(0,S,R\right) \end{array}$$

$$+\frac{1}{\mathfrak{m}\left(0,\mu\left(\cdot\right)\right)}\left(\frac{-\partial\mathfrak{m}\left(0,\mu\left(\cdot\right)\right)}{\partial\mu\left(\mathbf{x}_0\right)}\right).$$

$$\frac{1}{\mathfrak{m}\left(0,\mu\left(\cdot\right)\right)}\left(\frac{-\partial\mathfrak{m}\left(0,\mu\left(\cdot\right)\right)}{\partial\mu\left(\mathbf{x}_0\right)}\right)=\frac{-\int_{x_0}^T\mathbf{e}^{-\left[\left(1-\sigma\right)r+\sigma\rho\right]x}\left(\mathbf{x}\right)\,\mathrm{d}\mathbf{x}}{\int_0^T\mathbf{e}^{-\left[\left(1-\sigma\right)r+\sigma\rho\right]x}\left(\mathbf{x}\right)\,\mathrm{d}\mathbf{x}},$$

$$[0,\mathsf{T}]$$

$$\mathsf{T}$$

$$\frac{1}{\mathsf{H}\left(\mathsf{S}\,,\widetilde{\mathsf{R}}\left(\mathsf{S}\,,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}\frac{-\partial\mathsf{H}\left(\mathsf{S}\,,\widetilde{\mathsf{R}}\left(\mathsf{S}\,,\mu\left(\cdot\right)\right),\mu\left(\cdot\right)\right)}{\partial\mu\left(\mathbf{x}_0\right)}$$

$$=\frac{1}{\frac{\int_{x_0}^{R^*}e^{-rx}l(x)w(x)dx}{\int_{S^*}^{R^*}e^{-rx}l(x)w(x)dx}}\begin{array}{l} \mathbf{x}_0\leq \mathsf{S} \\ \mathsf{S}<\mathbf{x}_0<\mathsf{R} \\ \mathbf{x}_0\geq \mathsf{R} \end{array}.$$

$$0<\frac{\int_{x_0}^{R^*}e^{-rx}l(x)w(x)dx}{\int_{S^*}^{R^*}e^{-rx}l(x)w(x)dx}<1\qquad\mathbf{x}_0\qquad\qquad\qquad(\mathsf{S}\,,\mathsf{R}\,)$$

$$\mathfrak{ff}$$

$$\mathfrak{fi}$$

$$\mathbf{x}_0<\mathsf{R}$$

$$\mathfrak{ff}$$

$$\mathfrak{ff}$$

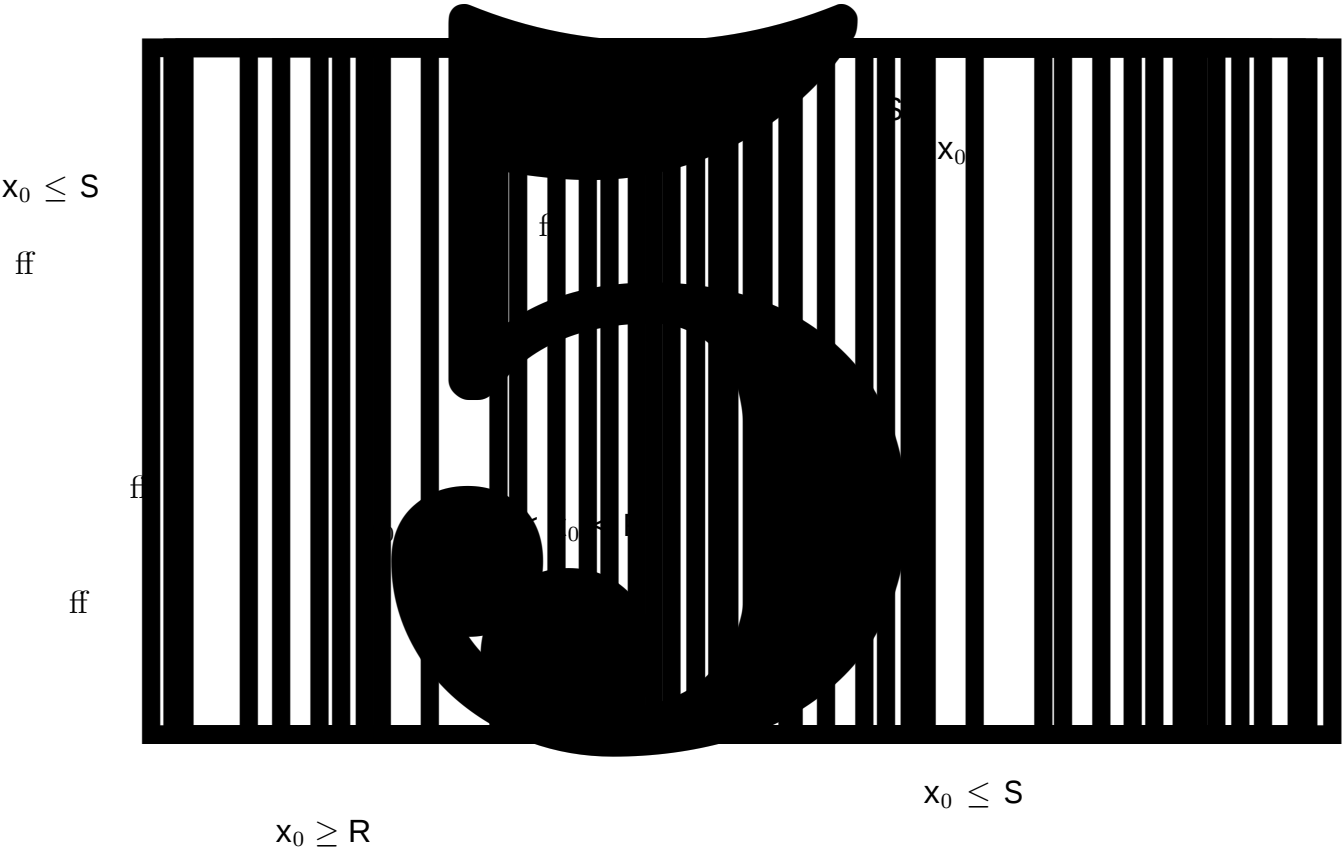
$$\mathrm{sign}\left[\frac{-\partial\widetilde{\mathsf{R}}\left(\mathsf{S}\,,\mu\left(\cdot\right)\right)}{\partial\mu\left(\mathbf{x}_0\right)}\right]=\begin{array}{ccc} & -1 & \\ -1 & 0 & +1 \\ & +1 & \end{array}\begin{array}{l} \mathbf{x}_0\leq \mathsf{S} \\ \mathsf{S}<\mathbf{x}_0<\mathsf{R} \\ \mathbf{x}_0\geq \mathsf{R} \end{array}.$$

$$\mathbf{x}_0\qquad\qquad\qquad\mathbf{x}_0\geq\qquad\qquad\qquad\mathfrak{ff}$$

$$\mathsf{R}$$

$$\mathfrak{ff}$$

$$\hline$$



$$\text{sign} \left[ \frac{-\partial S}{\partial \mu(x_0)} \right] = \text{sign} \left[ \frac{-\partial \tilde{R}(S, \mu(.))}{\partial \mu(x_0)} \right] = \begin{cases} -1 & x_0 \leq S \\ +1 & x_0 \geq R \end{cases}.$$

$$\text{ff} \quad \text{ff} \quad \text{ff}$$

$$S < x_0 < R$$

$$\text{sign} \left[ \frac{-\partial S}{\partial \mu(x_0)} \right] = \text{sign} \left[ \frac{\int_{x_0}^{R^*} e^{-rx} l(x) w(x) dx}{\int_{S^*}^{R^*} e^{-rx} l(x) w(x) dx} + \frac{e^{-rR^*} l(R^*) w(R^*) \left( \frac{-\partial \tilde{R}(S^*, \mu(.))}{\partial \mu(x_0)} \right)}{\int_{S^*}^{R^*} e^{-rx} l(x) w(x) dx} \right]$$

$$= -1, 0, +1.$$

ff ff ff

ff

 $x_0 \text{ ff}$ 
$$\frac{\partial \tilde{R}(S^*, \mu(\cdot))}{\partial S} \begin{matrix} \text{fi} & \text{ff} \\ \left( \frac{-\partial S^*}{\partial \mu(x_0)} \right) \end{matrix}$$

ff

$$\frac{-\partial \tilde{R}(S^*, \mu(.))}{\partial \mu(x_0)}$$
$$\text{sign} \left[ \frac{-\partial \tilde{R}(S^*, \mu(\cdot))}{\partial \mu(x_0)} \right]$$
$$\text{sign} \left[ \frac{-\partial S^*}{\partial \mu(x_0)} \right]$$
$$\sin \left[ \frac{\pi}{2} \left( \frac{S}{S_0} \right) \right] > 0.$$

ff

$$\text{sign} \left[ \frac{-\partial \tilde{R}(\mathbf{x}_0)}{\partial \mu(\mathbf{x}_0)} \right] = \text{sign} \left[ \frac{\partial \tilde{R}(\mathbf{x}_0)}{\partial \mu(\mathbf{x}_0)} \right] + \left( \frac{-\partial \tilde{R}(\mathbf{x}_0)}{\partial \mu(\mathbf{x}_0)} \right)$$
$$= \begin{matrix} & -1 \\ & ,0 \\ & 1 \end{matrix} \quad \begin{matrix} S \\ < R \\ R \end{matrix}$$

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S

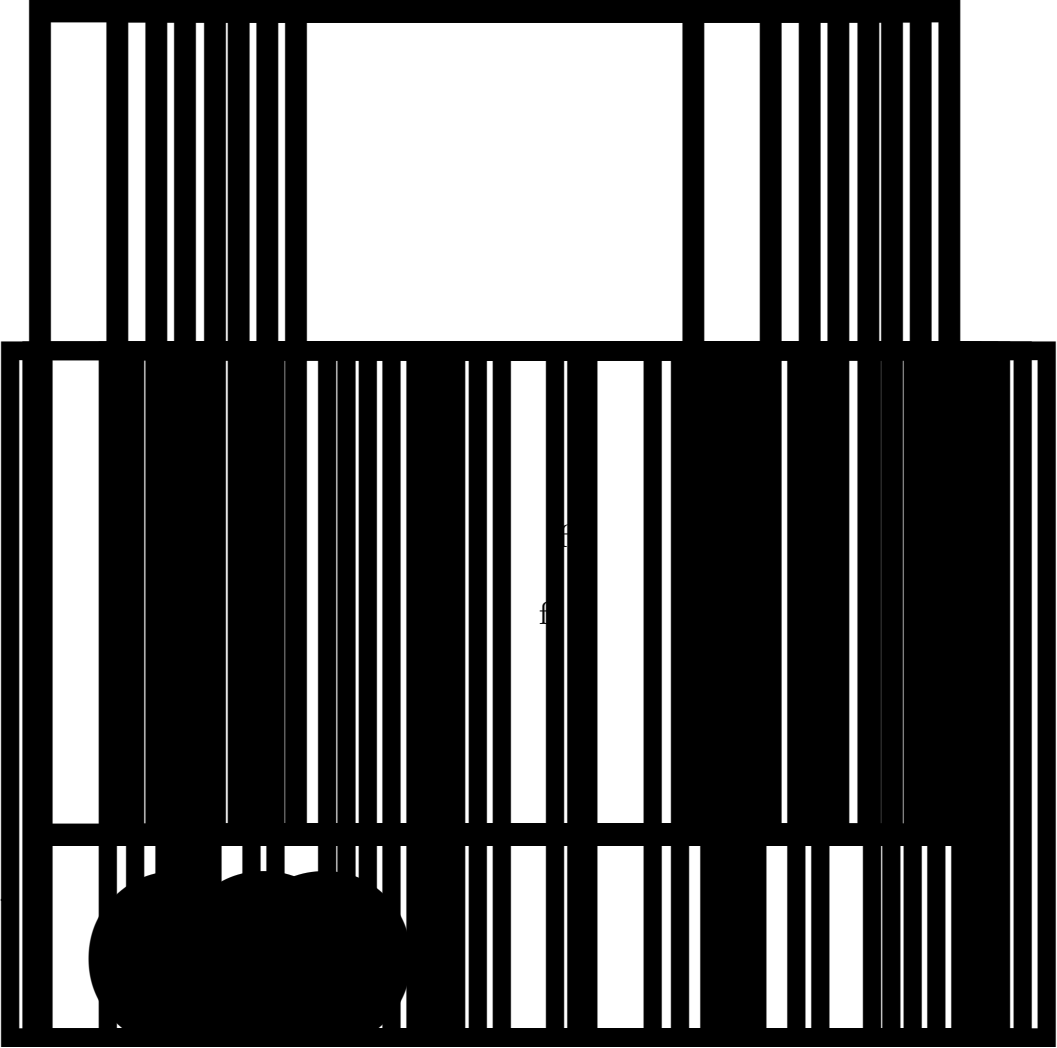
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$$\int_0^T e^{-rx}l\left(x\right)c\left(x,S,R\right)dx=\int_S^R e^{-rx}l\left(x\right)h\left(S\right)w\left(x\right)dx,$$

S R

$$\frac{\partial c\left(0,S,R\right)}{\partial S}=m\left(0\right)\left[h'\left(S\right)\int_S^R e^{-rx}l\left(x\right)w\left(x\right)dx-e^{-rS}l\left(S\right)h\left(S\right)w\left(S\right)\right],$$

$$\frac{\partial c\left(0,S,R\right)}{\partial R}=m\left(0\right)h\left(S\right)e^{-rR}l\left(R\right)w\left(R\right).$$

S

fi



S

R

$$U\left( S,R\right) =\int_0^Te^{-\rho x}I\left( x\right) \frac{c\left( x,S,R\right) ^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}dx-\int_0^Re^{-\rho x}I\left( x\right) f\left( x\right) dx.$$

$$\frac{\partial U\left( S,R\right) }{\partial R}$$

$$\frac{\partial U\left( S,R\right) }{\partial R}=e^{-rR}I\left( R\right) h\left( S\right) w\left( R\right) c\left( 0,S,R\right) ^{-\frac{1}{\sigma}}-e^{-\rho R}I\left( R\right) f\left( R\right) .$$

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$$I\left( R\right)$$

$$r-\rho+\frac{f'\left(\tilde{R}\left(S\right)\right)}{f\left(\tilde{R}\left(S\right)\right)}-\frac{w'\left(\tilde{R}\left(S\right)\right)}{w\left(\tilde{R}\left(S\right)\right)}+\frac{e^{-r\tilde{R}\left(S\right)}I\left(\tilde{R}\left(S\right)\right)w\left(\tilde{R}\left(S\right)\right)}{\sigma\int_S^{\tilde{R}\left(S\right)}e^{-rx}I\left(x\right)w\left(x\right)dx}>0$$

$$\frac{\partial^2U\left(S,R\right)}{\partial R^2}\Big|_{R=\tilde{R}\left(S\right)}<0$$

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$$V\left( S\right) =\int_0^Te^{-\rho x}I\left( x\right) \frac{c\left( x,S,\tilde{R}\left( S\right) \right) ^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}dx-\int_0^{\tilde{R}\left( S\right )}e^{-\rho x}I\left( x\right) f\left( x\right) dx.$$

$$\frac{\partial V\left( S\right) }{\partial S}$$

$$V'\left( S\right) =c\left( 0,S,\tilde{R}\left( S\right) \right) ^{-\frac{1}{\sigma}}\left[ h'\left( S\right) \int_S^{\tilde{R}\left( S\right )}e^{-rx}I\left( x\right) w\left( x\right) dx-e^{-rS}I\left( S\right) h\left( S\right) w\left( S\right) \right] .$$

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$$\frac{w'\left( S\right) }{w\left( S\right) }+\frac{2h'\left( S\right) }{h\left( S\right) }-\frac{h''\left( S\right) }{h'\left( S\right) }-\mu\left( S\right) -r-\frac{e^{-r\tilde{R}\left( S^{*}\right )}I\left(\tilde{R}\left(S\right)\right)w\left(\tilde{R}\left(S\right)\right)}{\int_{S^{*}}^{\tilde{R}\left(S^{*}\right)}e^{-rx}I\left(x\right)w\left(x\right)dx}\frac{\partial \tilde{R}\left( S\right) }{\partial S}>0$$

$$\text{fi}\quad V''\left( S\right) <0$$



$$\mathbb{E}\left[\frac{\partial \tilde{R}(S)}{\partial \mu(x_0)}\right] = \frac{\frac{-1}{\sigma c(0,S,\tilde{R}(S))} \frac{\partial c(0,S,\tilde{R}(S))}{\partial \mu(x_0)}}{\mathbf{r} - \rho + \frac{f'(\tilde{R}(S))}{f(\tilde{R}(S))} - \frac{w'(\tilde{R}(S))}{w(\tilde{R}(S))} + \frac{e^{-r\tilde{R}(S)}l(\tilde{R}(S))w(\tilde{R}(S))}{\sigma \int_S^{\tilde{R}(S)} e^{-rx}l(x)w(x)dx}}.$$

$$\mathbb{E}\left[\frac{\partial \tilde{R}(S)}{\partial S}\right] = \frac{\frac{h'(S)}{h(S)} - \frac{1}{\sigma c(0,S,\tilde{R}(S))} \frac{\partial c(0,S,\tilde{R}(S))}{\partial S}}{\mathbf{r} - \rho + \frac{f'(\tilde{R}(S))}{f(\tilde{R}(S))} - \frac{w'(\tilde{R}(S))}{w(\tilde{R}(S))} + \frac{e^{-r\tilde{R}(S)}l(\tilde{R}(S))w(\tilde{R}(S))}{\sigma \int_S^{\tilde{R}(S)} e^{-rx}l(x)w(x)dx}}.$$

$$\text{sign}\left[\frac{\partial \tilde{R}(S)}{\partial S}\right] = \text{sign}\left[\frac{h'(S)}{h(S)} - \frac{1}{\sigma c(0,S,\tilde{R}(S))} \frac{\partial c(0,S,\tilde{R}(S))}{\partial S}\right].$$

$$\frac{\partial c(0,S,\tilde{R}(S))}{\partial S} = 0.$$

$$\mathbb{E}\left[\frac{\partial \tilde{R}(S)}{\partial S}\right] = \frac{\frac{h'(S)}{h(S)} - \frac{1}{\sigma c(0,S,\tilde{R}(S))} \frac{\partial c(0,S,\tilde{R}(S))}{\partial S}}{\mathbf{r} - \rho + \frac{f'(\tilde{R}(S))}{f(\tilde{R}(S))} - \frac{w'(\tilde{R}(S))}{w(\tilde{R}(S))} + \frac{e^{-r\tilde{R}(S)}l(\tilde{R}(S))w(\tilde{R}(S))}{\sigma \int_S^{\tilde{R}(S)} e^{-rx}l(x)w(x)dx}}.$$



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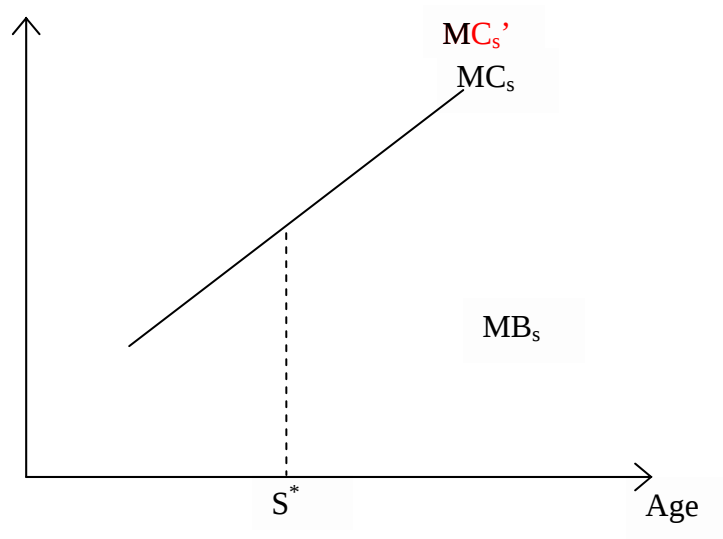
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|                                             | <b>Survival function</b> | <b>Is retirement age endogenous?</b> | <b>Elements of schooling cost</b> |
|---------------------------------------------|--------------------------|--------------------------------------|-----------------------------------|
| de la Croix and Licandro (1999)             | Exponential              | No                                   | Foregone earning                  |
| Hu (1999)                                   | Exponential              | No                                   | Foregone earning                  |
| Bils and Klenow (2000)                      | Rectangular              | No                                   | Foregone earning and tuition cost |
| Kalemli-Ozcan, Ryder and Weil (2000)        | Exponential              | No                                   | Foregone earning                  |
| Boucekkine, de la Croix and Licandro (2002) | BCL                      | Yes                                  | Foregone earning                  |
| Boucekkine, de la Croix and Licandro (2003) | BCL                      | No                                   | Foregone earning                  |
| Echevarria (2004)                           | Rectangular              | Yes                                  | Foregone earning                  |
| Echevarria and Iza (2006)                   | BCL                      | Yes                                  | Foregone earning                  |
| Hazan (2009)                                | Rectangular              | Yes                                  | Foregone earning                  |
| Heijdra and Romp (2009)                     | Gompertz-Makeham         | No                                   | Foregone earning                  |
| Cervellati and Sunde (2010)                 | General                  | No                                   | Foregone earning                  |

Note: BCL stands for the two-parameter specification used in Boucekkine, de la Croix and Licandro (2002).

**Table 1: Recent Papers on Mortality Decline and Schooling Years**



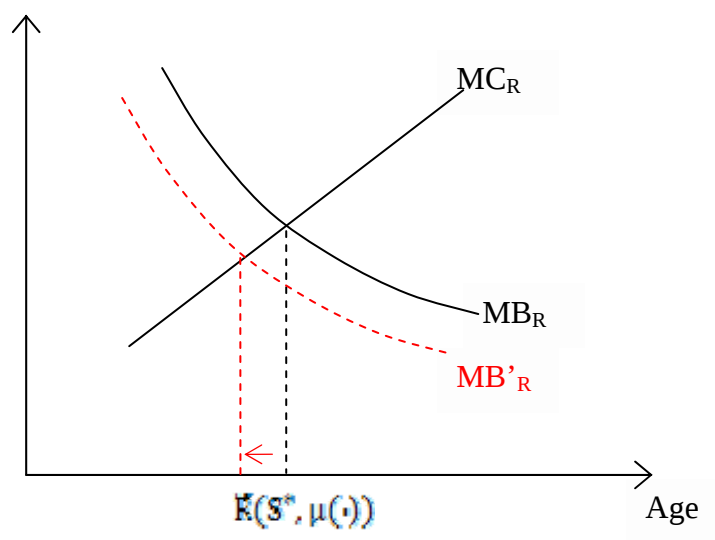
(a)  $x_0 \leq S^*$

$MC_s$  ( $MC_s'$ )

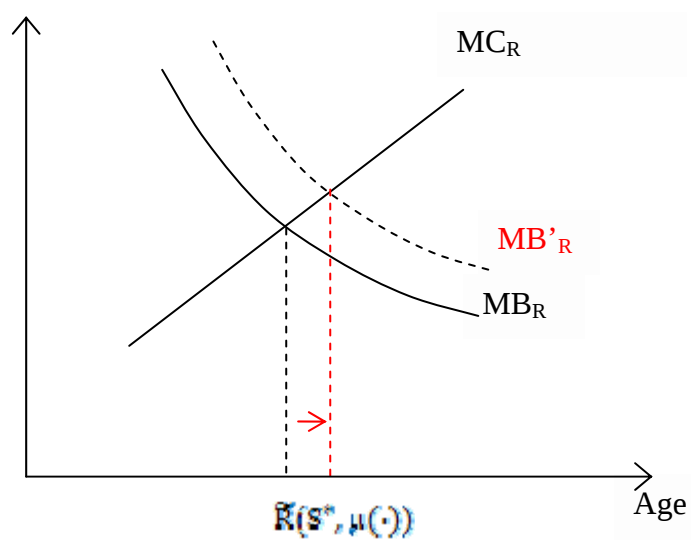
$MB_s$

Age

(b)  $S^* < x_0 < R^*$



(a)  $x_0 \leq S^*$



(b)  $x_0 \geq R^*$

**Figure 2: Indirect Effects on Schooling Years through Retirement Age**